

Markscheme

November 2020

Extended mathematics

On-screen examination



23 pages

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The markscheme may make use of the following abbreviations:

ECF Marks that can be awarded as error carried forward from previous results in the question BOD Benefit of the doubt MR misread NWS no working shown SC special case OE or equivalent WTTE or words to that effect AG Answer given

• Bullet notation means award 1 mark – see example 1 below



Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, Error Carried Forward (ECF) marks are awarded after an error.

- a) ECF applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or sinx > 1) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a simpler question then ECF may not be fully awarded.
- d) To award ECF marks for a question part, there must be working present for that part.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (**MR**) is an error. **ECF** is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation e.g x² for x² in final answers unless noted otherwise in the MS.
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- d) In the case when a correct result is obtained by coincidence based on incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- f) In the markscheme, equivalent examples of numerical and algebraic forms or simplified answers will generally be written in the notes preceded by OE or

equivalent e.g.
$$\frac{1}{2}$$
 or 1/2 or 1÷2 and $\frac{x}{2}$ or x/2 or x÷2

- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks SC can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) When a calculator screenshot is taken, accept not seeing the whole operation.
- j) Accept seeing an equation not in-line



mathemoeengtz0xxm

f	2cos ² θ	1
g	geometric sequence	1

- 6 -

Question		Answers	Notes	Total
2	а	$2\ln(x-6)$ OE		1
	b	•1 (s =) 2 •2 (t =) 6		2
	C	 1 correctly eliminate In 2 correctly expand their quadratic 3 correctly factorise their quadratic or substitute into quadratic formula 4 correctly select their single result after factorisation or using quadratic formula 	 1 (x - 6)²= x 2 their x² - 12x + 36 = x or their x² - 13x + 36 = 0 3 their (x - 4)(x - 9) 4 their (x =) 9 only, Accept only if their x >6 	4

Standard (9 marks)

Que	stion	Answers Notes		Total
3	а	 1 seeing 40(%) 2 multiply their 40 % by 25 % AG 0.1 	 •1 0.4, ACCEPT 40(%) on the diagram •2 0.4 x 0.25 OE 	
	b	Correctly write 3 as the answer		1
	c	 correctly white 3 as the answer correctly place their 3 and their 27 in 'First donor' correct probabilities for their O- second branches correct probabilities for their Not O- second branches 	First donor Second donor 2 0 - 29 0 - 29 Not 0 - 29 Not 0 - 29 Not 0 - 29 0 - 29	3
			.1 ACCEPT their 3 only if positive integer 0 <their3<30 .2 or .3 ACCEPT only 0<their probability<1<="" td=""><td></td></their></their3<30 	

d	•1 correctly calculate the probability first O – and second O – from their tree diagram	•1 $\frac{\text{their3}}{30} \times \frac{\text{their2}}{\text{their29}}$ or $\frac{1}{145}$ OE	
	•2 correctly calculate the probability first not O – and second O – from their tree diagram	•2 $\frac{\text{their}27}{30} \times \frac{\text{their}3}{\text{their}29}$ or $\frac{27}{290}$ OE	
	•3 correctly substitute their values into the correct conditional probability formula	•3 $\frac{\text{their} \cdot 2}{\text{their} \cdot 1 + \text{their} \cdot 2}$ OE	4
	•4 correctly calculate their probability after applying the formula	•4 $\frac{their27}{their29}$ OE	

Que	stion	Answers	Notes	Total
4 a		 1 correctly write 5 hours 15 mins as 5.25 (h) 2 correctly calculate their volume in cubic metres 3 correctly set their equation of v 4 correctly calculate their r² 5 correctly calculate their r before rounding AG 4 m 	•1 ACCEPT not seeing this step •2 5.25 x 11.2 or 58.8 or 58800/1000 seen •3 $\pi r^2 \times 1.17$ = their 58.8 •4 $\frac{58.8}{\pi \times 1.17}$ or 15.997() or 15.779() •5 3.99() or 3.97() Accept only if their answer rounds to 4 For .4 and .5 DO NOT ACCEPT substituting <i>r</i> = 4 and calculating volume	5
b •1 correctly substitute 4 into the correct area of circle formula		•1 correctly substitute 4 into the correct area of circle formula	•1 $\pi \times 16$ or $\pi \times 4^2$ or 50.27 or 50.24 (using 3.14)	
		•2 correctly substitute 4 into the correct circumference of a circle formula	•2 $\pi \times 8 \text{ or } 2 \times \pi \times 4 \text{ or } 25.1(327)$	
		•3 multiply their circumference of circle by 0.3	•3 their $\pi \times 8 \times 0.3$ or their $2 \times \pi \times 4 \times 0.3$ or 7.5(398)	5
		•4 correctly add their •1 and •3	•4 Their 57.8(05)	
		•5 correctly round their .4 to the nearest square metre	•5 their 58 ACCEPT earlier all correct rounding	
	С	correctly multiply their 57.8(05) or 58 by 3.4	Their (\$)196.52 or (\$)197.2	
			ACCEPT their 196.52 without working DO NOT ACCEPT their 57.8(05…) x 3.4 incorrectly rounded in c)	1

Questi	on	Answers	Notes	Total
5	а	•1 correctly determine 1.95	•1 12.55 – 10.6 1 95	
		•2 correct trig ratio used	•2 $\cos y = \frac{1000}{5.7}$ accept not seeing this step	3
		•3 correctly write their answer before rounding or correct inverse trig	•3 $y = \cos^{-1} \frac{1.95}{5.7}$ or 69.99(480991) Accept only if their answer	
		AG 70(°)	rounds to 70	
	b	AM1	AM1	
		 1 correctly convert 70 from degrees to radians 	•1 $70 \times \frac{\pi}{180}$ or $\frac{7\pi}{18}$ seen	
		•2 subtract double of y from π	•2 $\pi - 2 \times \frac{7\pi}{18}$	
		AM2		2
		 1 subtract double of y from 180 	AM2	
		2 correctly convert 40 from degrees to rediene	canvas in part (a)	
		•2 correctly convert 40 from degrees to radians		
		AG $\frac{2}{9}\pi$	•2 $40 \times \frac{\pi}{180}$	
	С	AM1	AM1	
		•1 correct trig ratio used with 10.6 OR correctly substitute into sine rule	•1 $\cos\frac{7\pi}{18} = \frac{10.6}{v}$ or $\sin\frac{\pi}{9} = \frac{10.6}{v}$ OE OR $\frac{v}{\sin\frac{7\pi}{18}} = \frac{21.2}{\sin\frac{2\pi}{9}}$ OE	
			ACCEPT using degrees: $\cos 70 = \frac{10.6}{v}$ OE or $\sin 20 = \frac{10.6}{v}$ or	
		 2 correctly rearrange their trig ratio for v 	$\frac{v}{\sin 70} = \frac{21.2}{\sin 40} \mathbf{OE}$	3
		•3 correctly calculate their v after dividing by their trig ratio	•1 ACCEPT 10.6 and $\cos \frac{7\pi}{18}$ or $\sin \frac{\pi}{9}$ seen	
			•2 their $(v =) \frac{10.6}{\cos 70}$ or their $(v =) \frac{10.6}{\sin 20}$ or their $(v =) \frac{21.2 \sin 70}{\sin 40}$ OE	

 AM2 1 correct trig ratio used with 12.55 OR correctly substitute into sine rule 2 correctly calculate their hypotenuse 3 correctly calculate their <i>v</i> after subtracting 5.7 from their calculated hypotenuse AM3 1 seeing evidence of using similar triangles correctly 	•3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded AM2 •1 $\cos \frac{7\pi}{18} \text{ or } \sin \frac{\pi}{9} = \frac{12.55}{hyp} \text{ OR } \frac{a}{\sin \frac{7\pi}{18}} = \frac{25.1}{\sin \frac{2\pi}{9}} \text{ OE}$ •2 their 36.69(37) ACCEPT (hyp=) $\frac{12.55}{\cos 70} \text{ OE}$ •3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded	
•3 correctly calculate their <i>v</i> after dividing their product	AM3 •1 Seeing $\frac{12.55}{1.95}$ or $\frac{10.6}{12.55}$ OE or $\frac{v}{10.6} = \frac{v+5.7}{12.55}$ OE •2 $10.6 \times \frac{5.7}{1.95}$ OR $1.95v = 60.42$ •3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded	

d	AM1	AM1	
	•1 correctly substitute $\frac{2}{9}\pi$ and their v or their v+5.7 into correct sector formula •2 subtract their areas of sectors	•1 $(A =)\frac{1}{2}$ their 36.7 ² $\times \frac{2}{9}\pi$ or $(A =)\frac{1}{2}$ their 31 ² $\times \frac{2}{9}\pi$ •2 $(A =)\frac{1}{2}$ their 36.7 ² $\times \frac{2}{9}\pi - \frac{1}{2}$ their 31 ² $\times \frac{2}{9}\pi$	
	 •3 correctly calculate their area after subtracting their areas of sectors AM 2 •1 correctly substitute their v or their v+5.7 into correct area of circle formula •2 subtract their areas of circles or sectors •3 correctly calculate their result after multiplying their difference of areas by their 40/360 or x/(2 π) 	•3 their 134.6() or 134.7() or 135 AM 2 •1 $\pi \times their 36.7^2$ or $\pi \times their 31^2$ seen •2 $\pi \times their 36.7^2$ - $\pi \times their 31^2$ OE •3 their 134.6() or 134.7() or 135	3
e	 1 correctly calculate the material wasted 2 correctly write their fraction of material wasted 3 correctly calculate their percentage AM1 1 correctly write their fraction of material used 2 Subtract their fraction from 1 3 correctly calculate their percentage 	•1 $(25.1 \times 7.6 - \text{their } 134.6 =)$ their 56.16 •2 $\frac{\text{their } 56.16}{\text{their } 25.1 \times 7.6}$ •3 their 29() (%) AM1 •1 $\frac{\text{their } 134.6}{\text{their } 25.1 \times 7.6}$ •2 1 - their 0.705() •3 their 29() (%)	3

Question		Answers	Notes	Total
6	a	 1 correctly drag three heights 2 correctly drag the other three heights 	26 24 22 setup 18 18 10 10 10 10 20 30 40 50 60 Annual exposure / (E) SC: All six bars correct and thinner award 1 mark	2
	b	A valid statement including both 1990 and 2016	DO NOT ACCEPT a comment on only one interval Ex: in 1990, 3 countries had E between 0 and 10 while in 2016 there are 8 Ex: Highest in 1990 is 17 while highest in 2016 is 24 ACCEPT Overall correct comment ex: Less particulates or less exposure in the air in 2016 than 1990 OE Comments including more than one interval. Ex: In 2016 no countries had E more than 40 but in 1990 there were	1
	C	 1 seeing two correct midpoints and two correct frequencies 2 Add their four products of midpoints and frequencies 3 Divide the sum of their products by 39 4 Correctly write their mean before rounding AG 15.3 (µg/m³) 	 Any two of 5,15,25,35 AND any two of 8, 24, 5, 2 seen 2 5 x 8 + 15 x 24 + 25 x 5 + 35 x 2 OR 595 <i>their</i> 595/39 4 15.25(6). ACCEPT only if their answer rounds to 15.3 	4
	d	●1 (a =) - 1/5 OE ●2 (b =) 18	 •1 ACCEPT - 0.21 < a < -0.14 •2 ACCEPT correctly calculated <i>b</i> from their correct <i>a</i> 	2

6	e	Mark	1	2	3	
		F Identification of Factors	One factor mentioned from: The model/equation used to predict. or Whether the data will follow same trend or not or Acknowledge the E is decreasing or The fact that these are models based on scatter plots. ACCEPT factors affecting the trend like more data in the last 5 years collected or using electric cars or new technology that affects the environment or using solar power instead of petrol or air pollution or control of particular matter DO NOT ACCEPT only saying many factors			10
		PL (Prediction from line)	Attempt to calculate the E in 2030 using their line equation Ex: Substitute 40 in the equation and make incorrect calculation OR Substitute 2030 into the equation and make correct calculation OR Attempt to use the drop every 5 years to predict the drop in 2030 from the graph OR Describe the trend in words and predict it will reach below 13	Correctly calculate the E in 2030 using their line equation Ex: their (-0.15(40)+18 =)12 OR correctly use the drop every 5 years to predict the drop in 2030 from the graph		

DE	Attempt to calculate the E in 2030	Correctly calculate the E in 2030 using the	
(Prediction	using the exponential equation	exponential equation	
from		$(6 \times 0.01^{40} + 12.8)$ (270) coport 14	
Exponential)	Ex: Substitute 40 in the equation	$(0 \times 0.91 + 13.0 =)13.9(379)$, accept 14	
	$(6 \times 0.01^{40} + 13.8)$ and make incorrect		
		AD	
	calculation		
	OK Cubatitute 2020 into the equation and	use the drop every 5 years to predict the	
	Substitute 2030 Into the equation and	drop in 2030 from the graph	
		ACCEPT if they mention that the	
	Attempt to use the drep overy E veers	ACCEPT in they mention that the	
	to predict the drop in 2030 from the	borizontal assymptote being E=13.8	
	araph		
	OR		
	Describe the trend in words and		
	predict it will not reach 13		
D	Inaccurate with weak reason	Inaccurate with a valid reason related to	
dearee of	(Interpolating)	variables affecting the future (extrapolating)	
accuracy	Example:	Ex:	
	inaccurate because I used line of best	The prediction I made not very accurate	
	fit	because many factors may vary in the	
	Inaccurate because of sudden	future	
	increase of pollution happening	or	
	OR	predictions using line of best fit for the	
	Accurate with valid reason	future not guaranteed	
	Example:	or	
	accurate because I used my line of	The prediction using the line equation not	
	best fit to estimate and made correct	very accurate because it is taking only a	
	calculations	window or isolated time	
	Rounding to a whole number used for	The prediction using the line equation not	
	their estimated year	very accurate because it assumes the	
	Ev: 14 for the E from expensetial	ruture follows same pattern	
	DO NOT ACCEPT if they just write	My predictions not yory accurate because a	
	down a year without any reference or	disaster may happen and affect pollution	
	calculations		
		DO NOT ACCEPT if they did not make a	
	DO NOT ACCEPT accurate or	prediction	
	inaccurate without reason		

	J	Selecting the Exponential model (even	Selecting the Exponential model (even	Selecting the Exponential model	
	Justifying	implicitly ex: substituting only in	implicitly) with general justification (no	(even implicitly)	
	the model	exponential model) without justification	explicit data)	With good justification involving	
		OR		data in the last years	
		Select the line with acceptable	Ex: I advise using graph 2 because it takes	-	
		justification Ex: it passes through more	most of the points into account or Line will	Ex: graph 2 is better because	
		points	decrease in a short time while exponential	in the last years, the E did not	
		OR	will take longer or exponential more fitting	vary much or it is nearly constant	
		comparison comment without		in the last years and it takes this	
		selection		into account while graph 1	
		Ex: Line will decrease in a short time		doesn't	
		while exponential will take longer OE		or the decrease in the line is	
				constant while E is not is not and	
				the exponential does not have	
				constant decrease rate	

	Answers	Notes	Total
a	AM1 •1 evidence of using Pythagoras correctly for diagonal of small square •2 correctly rearrange their equation to have y^2 on one side •3 square root the diagonal AM2 •1 evidence of using Pythagoras correctly in the right angled triangle •2 correctly rearrange their equation to have x^2 on one side •3 doubling the value of the side of triangle AM3 •1 evidence of using sin45 or cos45 •2 correctly rearrange their ratio to have unknown on one side •3 doubling the value of side of triangle AG $3\sqrt{2}$	AM1 •1 $3^2 + 3^2 = y^2$ •2 $18 = y^2$ •3 $\sqrt{18}$ AM2 •1 $x^2 + x^2 = 3^2$ •2 $x^2 = \frac{9}{2}$ •3 $2 \times \frac{3}{\sqrt{2}}$ AM3 •1 $\sin 45 = \frac{x}{3}$ or $\cos 45 = \frac{x}{3}$ or $x\sqrt{2} = 3$ OE •2 $(x =)3\sin 45$ or $(x =)3\cos 45$ or $(x =)\frac{3}{\sqrt{2}}$ OE •3 $2 \times 3\sin 45$ or $2 \times 3\cos 45$ or $(x =)\frac{2 \times 3}{\sqrt{2}}$ OE DO NOT AWARD • ³ unless their result simplifies to $3\sqrt{2}$ SC Using $(\frac{3\sqrt{2}}{2})^2 + (\frac{3\sqrt{2}}{2})^2 = 9$ and $\sqrt{9} = 3$ award 2 marks.	3
b	1 correctly place 1442 correctly place 288		2

С	•1 correctly describe one pattern in words	ACCEPT	
	•2 correctly describe a second pattern in words	Multiply by 2 every timeWTTE	
		All even numbers except first (9) or after stage 1	
	Allow ECF from their table in 8b	The ratio is constant	
		It is geometric	
		Multiplies of 9	
		Starting from 9, every second number is a square number	
		ACCEPT if they correctly describe in words their pattern	2
		DO NOT ACCEPT	
		It is the square of L	
		$A = L^2$	
		It is increasing	
		All A are even numbers	
		general rules in terms of n, example: $A = 9 \times 2^{n-1}$	
d	•1 The correct general rule	•1 $9 \times 2^{n-1}$ OE	
	•2 The correct general rule with correct notation	•2 $A = 9 \times 2^{n-1}$ OE	2
е	•1 correctly substitute $n \ge 5$ into their rule	•1 Ex: 9×2 ⁴	
	•2 correctly calculate their value of P after substitution $n \ge 5$	•2 Ex: 144 (for the n = 5)	
	•3 recognize that their result is the same as their predicted value	•3 Same as value I predicted in table (and we find the candidate has 144 in	
		the table for $n = 5$) OR same as when we continue the pattern and explains	
		how 144 is obtained from pattern of multiplying 72 by 2	
		•3 ACCEPT seeing the 144 in the table and seeing their calculated A=144	3
		when h = 5	
		SC for 1 mark	
		Correctly test by applying the steps of verification mentioned in the left	
		column with a value of $n \le 4$	
		SC for 1 mark	
		Correctly verify their described pattern or rule (e.g. recursive rule)	

7	7	f	Mark	1	2	3	4	
			Predictions (P)	Correctly predict one value for P	Correctly predict two values for P			
			Description (D)	Attempt to describe a pattern in words or to write a rule Ex: number beside $\sqrt{2}$ multiplies by 2 every other time $\sqrt{2}$ is there every other time OR Correctly describe one pattern in words for L OR Correctly describe in words their pattern for P OR Attempt to write down a general rule for L, example: $(\sqrt{2})^n$ OR Weak attempt to write down a general rule for P, example $(\sqrt{2})^{n-1}$ DO NOT ACCEPT L or P is increasing n goes up by 1	Correctly describe one pattern in words for P Ex: P multiplies by $\sqrt{2}$ P is a geometric sequence P is exponential function DO NOT ACCEPT the general rule in words OR Correctly write down general rule for L in terms of n. Rule: $(L =)3 \times (\sqrt{2})^{n-1}$ OR Acceptable attempt to write down a general rule for P, example: $12(\sqrt{2})^n$ or recursive rule For notation see N	Correctly describe one pattern in words for P AND Acceptable attempt to write down a general rule for P OR Correctly write down general rule for P in terms of n. $12(\sqrt{2})^{n-1}$ OE OR Correctly describe one pattern in words for P AND correctly write down general rule for L For notation see N	Correctly describe one pattern in words for P AND Correctly write down the general rule for P in terms of n For notation see N	20
			Testing (T)	Attempt to test their general rule for P using $n \le 4$ Ex: correctly substitute in their general rule value of $n \le 4$ Ex:	Correctly test their general rule for P using $n \le 4$ Ex: Correctly calculate their value for P in their general			

	substitute in their general rule value of n ≤ 4 OR	rule using $n \le 4$ AND Recognise that their correctly calculated value for P is the same as the given value.		
	Correctly test their described pattern or their rule (e.g. recursive rule)	ACCEPT seeing their correctly calculated value for P and the given value in the table being equal		
Verifying (V)	Attempt to verify their general rule for P using $n \ge 5$ Ex: correctly substitute in their general rule value of $n \ge 5$ OR Correctly verify their described pattern or their rule (e.g. recursive rule)	Orrectly calculate their value of P using their n ≥ 5 in their general rule	Correctly calculate their value for P in their general rule using n ≥5 AND Recognise that their correctly calculated value for P is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for P and their predicted value in the table being equal	
Justify/proof (J)	Weak attempt to justify their described pattern or their general rule Examples: trying at least two more values and arguing as justification that they are the same or rule works OR Just seeing their rule as $12 \times (\sqrt{2})^{n-1}$ without any explanation OR Assuming geometric sequence and valid attempt to find U1 and r	Good attempt to justify their general rule for P Examples: Recognise it is a geometric sequence and correctly write values of U1 and r OR multiplying by 4 incorrect rule for L	correctly justify the correct general rule for P in relation to geometry Examples: Writing in words OE that multiplying side by 4 gets perimeter if their L rule is $3 \times (\sqrt{2})^{n-1}$ and their general rule is $12 \times (\sqrt{2})^{n-1}$ J3 automatically gains T2 and V3	

	DO NOT ACCEPT if D2 not achieved			
Notation and terminology (N)	Correct notation of <u>their</u> rule OR Correct terminology describing a pattern DO NOT ACCEPT if they don't have any rules and they don't describe any patterns	Correct notation of <u>the general</u> rule for P OR The notation of <u>the general</u> rule includes errors AND Correct terminology describing pattern in words for P DO NOT ACCEPT if they don't have a general rule	Correct notation of <u>the general</u> rule for P AND Correct terminology describing pattern in words for P ACCEPT using Un instead of P only if they mention that P = Un For notation of the general rule, DO NOT ACCEPT Ex: $12(\sqrt{2})^{n}(n-1)$ Or the rule is: $12(\sqrt{2})^{n-1}$ Or non-simplified rules	
Communication (L)	Very weak communication More than two lines of communication that lacks organisation and coherence OR Only calculations or algebraic steps	Weak communication At least three lines of communication with an attempt of organisation but lacks coherence	Good communication More than three lines of communication with acceptable organisation and coherence Can be awarded only if J2 is achieved	

Stage number (n)	Side length (L)	Perimeter of square (P)
1	3	12
2	3√2	12√2
3	6	24
4	6√2	24√2
5	12	48
6	12√2	48√2
7	24	96
8		

General rules:

 $L = 3 \times (\sqrt{2})^{n-1} \text{ or } 3 \times 2^{\frac{n}{2} - \frac{1}{2}}$ $P = 12 \times (\sqrt{2})^{n-1} \text{ or } 3 \times 2^{\frac{n}{2} + \frac{3}{2}}$